Equilibrium Price Formation in Markets with Differentiated Informed Agents

Marta Faias
Universidade Nova de Lisboa.
e-mail: mcm@fct.unl.pt

Carlos Hervés-Beloso
RGEA. Univerrsidad de Vigo.
e-mail: cherves@uvigo.es

Emma Moreno-García
Universidad de Salamanca.
e-mail: emmam@usal.es

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Corresponding author: Emma Moreno-García. Facultad de Economía y Empresa. Universidad de Salamanca. Campus Miguel de Unamuno. 37007 Salamanca. SPAIN. Fax:+34-923-294686. e-mail: emmam@usal.es

Abstract. We consider a pure exchange economy with a finite set of types of agents which have incomplete and asymmetric information on the states of nature. Our aim is to describe the equilibrium price formation and analyze how the lack of information may affect the allocation of resources. For it, we adapt to an asymmetric information scenario a variant of the Shapley-Shubik game introduced by Dubey and Geanakoplos (2003).

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1 Introduction

We consider a pure exchange economy with a finite number of types of agents and commodities. The economy extends over two periods and agents arrange contracts at the first period that may be contingent on the realized state of nature in the second period. Agents have incomplete information on a finite set of the states of nature and this information may differ among agents (differential information economies). After the realization of the state of nature, a particular agent may be unaware of the precise state of nature that has actually occurred, (since, for example, she receives a signal that may be identical for different states). Therefore they are restricted to sign contracts that are compatible with their private information.

For these economies, following an analogous concept to the Walrasian equilibrium in Arrow-Debreu model with symmetric information, Radner (1968, 1979) defined and established the existence of two different notions of equilibrium; Walrasian expectation or Radner equilibrium and Rational expectation equilibrium, respectively, depending on whether agents are able to learn from the prevailing price system. More recently, there has been a resurgent interest on differential information economies, and, in parallel to the Arrow-Debreu model, new results concerning the existence and characterization of equilibrium has been obtained (see Allen and Yannelis, 2001, Einy, et at al., 2001, Hervés-Beloso, et al., 2005a and 2005b, Daher,W., et al., 2006, Correia da Silva and Hervés-Beloso, 2009, De Castro and Yannelis, 2010).

Our aim is to use a market-game approach to study the behavior of these markets and to analyze the price formation mechanism.

The wide literature on market games uses the principles of game theory to motivate or justify the description of markets in which certain behavioral characteristics, such us price-taking behavior, are assumed. Most of these works show how strategic interactions by rational agents lead to a competitive equilibrium situation. One of the advantages of building a strategic foundation for perfect competition is that we will be forced to describe the process completely and explain how the market equilibrium is reached.

In order to explain the equilibrium price formation, we adapt a variant of the Shapley-Shubik game introduced by Dubey and Geanakoplos (2003). We describe new rules for price formation and the corresponding allocations, which underlie the differentiated information structures. For it, we define a market game that provides a mechanism for the equilibrium price formation and allows to observe how the lack of information may affect the allocation of wealth.

In our market game the mechanism leads consumers to receive the same bundle in states which they do not distinguish. Actually, the game forces consumers to be conservative or prudent (see Correia da Silva and Hervés-Beloso 2009, and De Castro and Yannelis 2010) and, as agents are familiar with the game, they will not be deceived. Then, without the Radner (1968) explicit assumption that agents choose constant consumption in states which they do not distinguish, this market game leads to a Radner equilibrium.

Moreover, we also describe a different game where the mechanism specifying the price formation leads to a particular price system which is compatible with the common information structure and we refer to as non-disclosure prices.

We show existence of Nash equilibrium for both types of games. Then, the corresponding market equilibrium solutions are obtained as a limit of a sequence of Nash equilibria. These limit results provide an alternative proof of the existence of Walrasiam Expectation Equilibrium (Radner 1968) and a proof for the existence of the refinement of equilibrium with prices that do not add any new insight to the private information of any agent.

For the general case our assumptions are the same as in Radner (1968). However, as we show in an example, in order to obtain the existence of the refinement of equilibrium, additional assumptions become necessary.

Regarding related work, Fugarolas et al. (2009) also undertake a non-cooperative approach to differential information economies by extending Schmeidler's (1980) work to the differential information setting. However, as in Schmeidler's result, the existence of Nash equilibrium is obtained as a consequence of the existence of the Walrasian expectations equilibrium and, since prices are included in the strategy sets, no explicit price formation rule is obtained.

The remaining of the paper is organized as follows. In Section 2 we describe the differential information model and the notion of market equilibrium. In Section 3 we state an associated game à la Shapley-Shubik and we prove existence of Nash equilibrium. In Section 4 we prove that the limit of a sequence of Nash equilibria results in a Radner equilibrium. A new game where the price resulting from the interaction among consumers are compatible with the common information structure is defined in Section 5. This game allows us to show existence of non-disclosure prices equilibrium. Finally, the last section is devoted to the conclusions.

2 The model

Let us consider an economy $\mathcal{E}$ with differential information. Let Ω be the set of states of nature that describes the uncertainty. We suppose that Ω is finite with cardinality k and there is a finite number of goods, L, in each state. There is a continuum of agents that trade the L commodities at each state of nature $\omega \in \Omega$.

The private information structure of each agent is described by a partition of the set of states Ω. Given a partition $\mathcal{P}$ of Ω, a commodity bundle $x = (x(\omega))_{\omega\in\Omega} \in (\mathbb{R}^L_+)^k$ is said to be $\mathcal{P}$-measurable when it is constant on the elements of the partition .[1]

The set of agents is represented by the unit real interval $I = [0,1] = \bigcup_{i=1}^{n} I_i$,[2] where $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $I_n = \left[\frac{n-1}{n}, 1\right]$. We consider the Lebesgue measure μ on the Borel subsets of I. Each agent $t \in I_i$ is characterized by her private information $\mathcal{P}_t = \mathcal{P}_i$, her initial endowments $e_t = e_i \in \mathbb{R}^{Lk}_+$ and preference relation over the consumption space, which is represented by a utility function $U_t = U_i : \mathbb{R}^{Lk}_+ \to \mathbb{R}_+$. We will refer to agents belonging to the subinterval I_i as agents of type i.

The economy lasts for two periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty about the states of nature and the agents make contracts (agreements) that are contingent on the realized state of nature that occurs at $\tau = 1$. Thus, the contracts are specified ex-ante.

An agent $t \in I_i$ with information given by the partition $\mathcal{P}_i$ is not able to distinguish those states of nature that are in the same element of $\mathcal{P}_i$. Given a state $\omega \in \Omega$, let $E_i(\omega)$ denote the event in the partition $\mathcal{P}_i$ which contains the state ω. We say that a consumption bundle $x \in (\mathbb{R}^L_+)^k$ is compatible with the

[1] That is, $x(\omega) = x(\omega')$, for all $\{\omega, \omega'\} \subseteq S$, for some $S \in \mathcal{P}$.

[2] There is a continuum of agents represented by the real interval $[0,1]$. Note that only a finite number of initial endowments and preference relations can be distinguished in the analysis. This implies that a finite number of different types of agents can be considered. Let us suppose that the measure of the set of agents of type j is a rational number $r_j/n,\ j = 1, \ldots, m$. Then we will consider n types of agents each one represented by the interval $\left[\frac{j-1}{n}, \frac{j}{n}\right)$ (observe that we will have r_j types of agents that are equal to agents of type j), with $j = 1, \ldots, n$.

information of agents of type i if, given any state ω, we have $x(\omega) = x(\omega')$ for every $\omega' \in E_i(\omega)$. Let us denote by $\mathcal{X}_i$ the set which consists in the bundles that are compatible with the information structure of agents of type i. That is,

$$\mathcal{X}_i = \left\{ x \in (\mathbb{R}^L_+)^k | x \text{ is } \mathcal{P}_i\text{-measurable} \right\}.$$

We state the following assumptions:

(U) For every i the utility U_i is a continuous, concave and monotone[3] function.

(E) $e_i \gg 0$ and $e_i \in \mathcal{X}_i,\ i = 1, \ldots, n$. That is, every agent is initially endowed with strictly positive amounts of every commodity and e_i is $\mathcal{P}_i$-measurable for every type i of consumers.

An allocation x is a μ-integrable function that associates to each agent t a consumption bundle x_t. We refer to an allocation x as *physically feasible* if $\int_I (x_t - e_t) d\mu(t) \leq 0$, and as *informationally feasible* if $x_t \in \mathcal{X}_i$, for every $t \in I_i$ and every i. A *feasible allocation* is both physically and informationally feasible.

Each agent $t \in I$ behaves as a price-taker and maximizes her utility function restricted to the allocations in her budget set. Given a price system $p \in \mathbb{R}^{Lk}_+$ that specifies a commodity price $p(\omega) \in \mathbb{R}^L_+$ at each state $\omega \in \Omega$, the budget set of an agent of type i is given by

$$B_i(p) = \{x \in \mathcal{X}_i \mid \sum_{\omega \in \Omega} p(\omega) \cdot (x(\omega) - e_i(\omega)) \leq 0\}.$$

Next we define a competitive equilibrium notion in the sense of Radner where traders must balance the budget ex-ante.

Definition 2.1 *A pair (p, x), where p is a price system and x is a feasible allocation, is a competitive or a Radner equilibrium if the bundle x_t maximizes U_t on $B_t(p)$, for almost all $t \in I$.*

Notice that we assume free disposal. It is well known that if we impose the condition of non-free disposal then a Radner equilibrium might not exist with positive prices (see, for example, Glycopantis, Muir and Yannelis, 2003).

[3] $x \gg y$ implies $U_i(x) > U_i(y)$

However, allowing for negative prices one can dispense with the free disposal assumption.

Finally, given our atomless economy $\mathcal{E}$, let us consider an economy $\mathcal{E}_n$ with a finite number n of agents. In the differential information economy $\mathcal{E}_n$ each agent i is characterized by an initial endowments e_i, the utility funtion U_i and a private information structure given by the partition $\mathcal{P}_i$. We have that if (p, x) is a competitive equilibrium for the continuum economy $\mathcal{E}$ then (p, z) is a competitive equilibrium for $\mathcal{E}_n$, where the allocation $z = (z_i, i = 1, \ldots n)$ is given by $z_i = \dfrac{1}{\mu(I_i)} \displaystyle\int_{I_i} x_t d\mu(t)$. Reciprocally, if (p, z) is a competitive equilibrium for the economy $\mathcal{E}_n$ with n consumers, then (p, x) is a competitive equilibrium for $\mathcal{E}$ where x is the step function given by $x_t = z_i$ for every consumer $t \in I_i$. Therefore, if we consider an economy with n consumers associated to the n-type continuum economy then the equilibrium solutions for the continuum and the discrete approach are equivalent (see Hervés-Beloso et. al, 2005, for details)

3 An Associated Game à la Shapley-Shubik

Following Shapley-Shubik (1977) approach, each commodity in each state of nature is traded at a trade-post, so there is a post for each commodity in each state of nature. Each consumer delivers to the post the endowment of commodity ℓ in each state $\omega \in \Omega$ for sale and fiat money to purchase the consumption goods and then the trading-post for commodity ℓ in the state ω receives the corresponding total endowment in the economy, i.e., $e^\ell(\omega) = \displaystyle\int_I e_t^\ell(\omega) d\mu(t)$. As in Dubey and Geanakoplos (2003) inside fiat money is the sole medium of exchange, agents initially have no money but can borrow up to a bound M at zero interest from a bank that, in order to trigger the market, places 1 unit of fiat money at each post. The trading-posts and the bank are dummy players. They can make no choices and therefore, they do not optimize.

In our scenario, once the total endowments are placed in the corresponding trading-posts, the game starts. The individuals choose strategies that precise the amount of fiat money to purchase the consumption goods.

The strategic variable of each agent is the amount of fiat money that she wants to spend in each contingent commodity. Precisely, to purchase commodity ℓ at the state ω each agent t delivers to the post fiat money $\theta_t^\ell(\omega)$ that she borrows at zero interest. In order to have compact strategy sets we impose an upper bound on borrowing. Thus, as we have already remarked, each agent cannot borrow more than M units of fiat money. Therefore, the strategy set of each consumer $t \in I$ is given by the set

$$\xi_t(M) = \xi(M) = \{\theta \in \mathbb{R}_+^{Lk} \text{ such that } \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \theta^\ell(\omega) \leq M\}.$$

A strategy profile $\Theta = (\theta_t, t \in I)$ is a μ-integrable function which associates to each player t a strategy θ_t.

We remark that, in this setting, the role of money is just a "means of payment" and prices are determined by the actions of traders. Given a strategy profile $\Theta = (\theta_t, t \in I)$, the price for each commodity ℓ in each state of nature $\omega \in \Omega$ arises in each post according to the next rule:

$$p^\ell(\omega) = \frac{\theta^\ell(\omega) + 1}{e^\ell(\omega)} > 0,$$

where $\theta^\ell(\omega) = \displaystyle\int_I \theta_t^\ell(\omega) d\mu(t)$.

Let $p(\Theta) = (p^\ell(\omega),\ \omega \in \Omega,\ \ell = 1, \ldots, L)$.

Let us consider an agent $t \in I_i$, and recall that $E_i(\omega)$ denote the event in the partition $\mathcal{P}_i$ which contains the state ω. The amount of commodity ℓ assigned to an individual $t \in I_i$ in the state ω is given by:

$$x_t^\ell(\omega) = \min\left\{\frac{\theta_t^\ell(\omega')}{p^\ell(\omega')},\ \omega' \in E_i(\omega)\right\}.$$

Let $x_t(\Theta) = (x_t^\ell(\omega),\ \omega \in \Omega,\ \ell = 1, \ldots, L)$ be the bundle allocated to consumer t when Θ is the strategy profile.

The agent $t \in I_i$ also receives money from the sale of her endowment, thus, his net deficit is given by,

$$d_t(\Theta) = \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \theta_t^\ell(\omega) - \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^\ell(\omega) e_t^\ell(\omega).$$

The payoff of each agent $t \in I_i$ for each strategy profile Θ is

$$\Pi_t(\Theta) = U_i(x_t(\Theta)) - d_{t+}(\Theta),$$

where $d_{t+} = \max\{0, d_t\}$. The use of the maximum to define the payoff function means that agents do not ascribe utility to fiat money, but are penalized in the case of default.

Now, let us show that the mechanism guarantees that for every strategy profile the resulting allocation of commodities is feasible. Note that the informational feasibility trivially follows since for each agent the mechanism assigns the same bundle in states that belong to the same event of the private partition. The final allocation for agents is physically feasible, in fact, for every commodity $\ell = 1, \ldots, L$ and every state $\omega \in \Omega$, the following inequality holds

$$\int_I x_t^\ell(\omega) d\mu(t) \le \int_I \frac{\theta_t^\ell(\omega)}{p^\ell(\omega)} d\mu(t) = \int_I \frac{\theta_t^\ell(\omega)}{\theta^\ell(\omega) + 1} e^\ell(\omega) d\mu(t) \le e^\ell(\omega).$$

Let $\mathcal{G}(M) \equiv \{(\Pi_t, \xi_t(M)) = \xi(M), t \in I\}$ denote the pseudo-game (to simplify, in which follows, we will just say game) previously described. Given a strategy profile $\Theta : I \to \xi(M)$ we denote by $\Theta \setminus \alpha_t$ the strategy profile which coincides with Θ except for player t who chooses α_t instead of $\Theta(t)$. A strategy profile Θ is a Nash equilibrium in the game $\mathcal{G}(M)$ if for almost all $t \in I$ we have $\Pi_t(\Theta) \ge \Pi_t(\Theta \setminus \alpha_t)$ for all $\alpha_t \in \xi(M)$.

Before showing a Nash equilibrium existence result for the game $\mathcal{G}(M)$, we state a Lemma obtaining a property of these equilibria that will be used in the convergence result presented in the next section.

Lemma 3.1 *If the profile $\Theta = (\theta_t, t \in I)$ is a Nash equilibrium for the game $\mathcal{G}(M)$, then for every commodity ℓ and type i we have $\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} = \dfrac{\theta_t^\ell(\bar{\omega})}{p^\ell(\bar{\omega})}$ for any $\omega \in E_i(\bar{\omega})$ for almost all $t \in I_i$.*

Proof. Assume that the statement of the Lemma does not hold. Then there exist a Nash equilibrium $\Theta = (\theta_t, t \in I)$ and a positive measure set J of agents of a type j such that, for every $t \in J \subset I_j$ one has $\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \neq \dfrac{\theta_t^\ell(\bar{\omega})}{p^\ell(\bar{\omega})}$ for some commodity ℓ and some states ω and $\bar{\omega}$ such that $\omega \in E_j(\bar{\omega})$[4]. For each $t \in J$, and each commodity ℓ let A_t^ℓ be the set of states at which the minimum of $\left\{\dfrac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar{\omega})\right\}$ is attained. Recall that one player is not able to alter

[4]The commodity ℓ and the states ω and $\bar{\omega}$ may depend on t.

the price by modifying her strategy unilaterally. For each $t \in J$ let us consider a strategy α_t given by

$$\alpha_t^\ell(\omega) = \begin{cases} \theta_t^\ell(\omega) \text{ if } \omega \text{ does not belong to } E_j(\bar{\omega}) \\ \theta_t^\ell(\omega) - \varepsilon_t \text{ if } \omega \text{ does not belong to } A_t^\ell \\ \theta_t^\ell(\omega) + \delta_t \text{ if } \omega \text{ belongs to } A_t^\ell \end{cases}$$

We can choose $\varepsilon_t > 0$ and $\delta_t > 0$ in such a way that $d_t(\Theta) \geq d_t(\Theta \setminus \alpha_t)$[5] and

$$\min\left\{\frac{\alpha_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar{\omega})\right\} > \min\left\{\frac{\theta_t^\ell(\omega)}{p^\ell(\omega)} \text{ with } \omega \in E_j(\bar{\omega})\right\}.$$

Therefore, this indicates the way to choose the strategy that gives to every player $t \in J$ an incentive to deviate from the profile Θ which is a contradiction to the conditions of Nash Equilibrium.

Q.E.D.

Observe that at the end of the previous proof we require the assumption of weak monotonicity as stated in the footnote 3. On the other hand, note that the Lemma guarantees that if Θ is a Nash equilibrium, then the resulting allocation $x(\Theta)$ is μ-integrable and therefore it is feasible.

A strategy profile Θ is called symmetric if every agent of the same type selects the same strategy, that is, $\Theta(t) = \theta_i$ for every $t \in I_i$. If it is the case, we write $\Theta = (\theta_1, \ldots, \theta_n) \in (\xi(M))^n$.

Theorem 3.1 *For every $M \in \mathbb{R}_+$ the set of symmetric Nash equilibria for the game $\mathcal{G}(M)$ is non-empty.*

Proof. Let $\mathcal{B}_t$ be a correspondence which associates to each symmetric strategy profile the best reply of the player $t \in I$. That is, given the strategy profile $\Theta = (\theta_1, \ldots, \theta_n) \in (\xi(M))^n$

$$\mathcal{B}_t(\Theta) = \arg\max_{\alpha_t \in \xi(M)} \Pi_t(\Theta \setminus \alpha_t)$$

[5]Let A_t denote the set of states in $E_j(\bar{\omega})$ which do not belong to A_t^ℓ and # denotes cardinal of the corresponding set. Then, we can take δ_t and ε_t in such a way that $\delta_t \# A_t^\ell \leq \varepsilon_t \# A_t$

Note that, by symmetry, $\mathcal{B}_t$ is the same for every player $t \in I_i$ and we denote $\mathcal{B}_i$. By definition, $p(\Theta) = p(\Theta \setminus \alpha_t)$ which allows us to obtain that $x_t(\Theta \setminus \alpha_t)$ is concave in α_t and $d_t(\Theta \setminus \alpha_t)$ is linear in α_t. Then, by assumption (U), we have that the payoff function Π_t is concave in the strategy selected by player t. This implies that $\mathcal{B}_t$ takes non-empty-convex values since $\xi(M)$ is a convex and compact set.

Moreover, the payoff function Π_t is a continuous function. Then, the maximun theorem allows us to conclude that the correspondence $\mathcal{B}_i$, from $(\xi(M))^n$ to $\xi(M)$, is upper semi-continuous for every $i = 1, \ldots, n$.

Finally, let us consider the correspondence $\mathcal{B} = (\mathcal{B}_1, \ldots, \mathcal{B}_n)$. By Kakutani's theorem $\mathcal{B}$ has a fixed point which is a symmetric Nash equilibrium.

Q.E.D.

4 Radner equilibrium as a limit of a sequence of Nash equilibria

In this section, we show that a Radner equilibrium can be obtained as the limit of a sequence of prices and allocations resulting from the sequence of symmetric Nash equilibria of the games $\mathcal{G}(M)$ when M goes to infinity. For it, given a price system $p \in \mathbb{R}_+^{Lk}$, let $\|p\| \equiv \sum_{\omega \in \Omega} \sum_{\ell=1}^{L} p^\ell(\omega)$.

Theorem 4.1 *For each integer M, let $\Theta_M = (\theta_{M,t},\ t \in I)$, be a symmetric Nash equilibrium for the game $\mathcal{G}(M)$. Let $(p(M), x(M))$ be the corresponding sequence of prices and allocations which is defined by the sequence of Nash equilibria. Then, there exists a subsequence of $(p(M)/\|p(M)\|, x(M))$ which converges to a price system p and an allocation x, such that (p, x) is an equilibrium for the economy $\mathcal{E}$.*

Proof. Since $\Theta_M = (\theta_{M,t},\ t \in I)$ is a symmetric Nash equilibrium for the game $\mathcal{G}(M)$, we have $\theta_{M,t} = \theta_{M,i}$ for every $t \in I_i$ and every type i of players. This equilibrium defines the price vector $p(M) = (p_M^\ell(\omega), \ell = 1, \ldots, L,\ \omega \in \Omega)$ which leads to the allocation $x(M) = (x_i(M),\ i = 1, \ldots, n)$ and net deficits $(d_i(M), i = 1, \ldots, n)$.

The definition of the game ensures that

$$\int_I x_t(M)d\mu(t) = \sum_{i=1}^n \mu(I_i)x_i(M) = \frac{1}{n}\sum_{i=1}^n x_i(M) \leq e = \sum_{i=1}^n \mu(I_i)e_i = \frac{1}{n}\sum_{i=1}^n e_i.$$

Thus, the consumption bundles allocated to consumers are bounded.

Note that if a player selects the strategy $\theta = 0$ then she spends and consumes nothing. This implies that $U_i(ne) - d_{i+}(M) \geq U_i(x_i(M)) - d_{i+}(M) \geq U_i(0)$ and then $d_{i+}(M)$ is bounded from above by $U_i(ne) - U_i(0)$.

Now, for each M let us consider the sets of types defined as follows:

$$D(M) = \{i \in \{1, \ldots, n\} \text{ such that } d_i(M) > 0\} \quad \text{and}$$

$$S(M) = \{i \in \{1, \ldots, n\} \text{ such that } d_i(M) < 0\}.$$

That is, $D(M)$ is the subset of types agents who are in deficit and $S(M)$ is the set of agents who are in surplus. It trivially holds the next equality

$$\sum_{i=1}^n d_i(M) = \sum_{i \in D(M)} d_i(M) + \sum_{i \in S(M)} d_i(M).$$

On the other hand, $0 = Lkn + \sum_{i=1}^n d_i(M) = Lkn - \sum_{i \in S(M)} -d_i(M) + \sum_{i \in D(M)} d_i(M)$, which implies that $\sum_{i \in S(M)} -d_i(M) = Lkn + \sum_{i \in D(M)} d_i(M)$ is also bounded from above. Since $d_{i+}(M)$ is bounded it follows that $-d_i(M)$ is also bounded. Finally, we can conclude that $d_i(M)$ is bounded.

Thus if we consider a sequence $(x_i(M), d_i(M), i = 1, \ldots, n)_M$ with M converging to infinity, there exists a converging subsequence with limit $(x_i, d_i, i = 1, \ldots, n)$. We write $x_i(M) \to x_i$ and $d_i(M) \to d_i$, for each type i. Moreover, the sequence $\frac{p(M)}{\|p(M)\|}$ has also a convergent subsequence with limit p. We write, $x_i(M) \to x_i,\ d_i(M) \to d_i$, for each type i, and $\frac{p(M)}{\|p(M)\|} \to p$.

We remark that since $x_i(M)$ belongs to $\mathcal{X}_i$ for every M and every i, the limit allocation x is informationally feasible, that is, x_i is $\mathcal{P}_i$-measurable for every type i of agents. It remains to show that $(p, (x_t)_{t \in I})$, with $x_t = x_i$ for every $t \in I_i$ is a Radner equilibrium.

Since $Lkn > 0$, the set $S(M)$ is nonempty. Moreover, every agent of type i in $S(M)$ must bid all the money that she can borrow. Otherwise, she could increase the bidding in every commodity in each state of nature which entails a strict increase in the consumption quantities of her bundle without incurring any default, and in consequence her payoff will increase. This is in contradiction with the fact that we are in a Nash equilibrium. Since any agent of type $i \in S(M)$ is in surplus we have that $p(M)e_i > M$ which implies $\|p(M)\| \to \infty$ when $M \to \infty$.

Recall that the allocation $x(M)$ is defined as

$$x^{\ell}_{M,i}(\bar{\omega}) = \min\left\{\frac{\theta^{\ell}_{M,i}(\omega)}{p^{\ell}_{M}(\omega)}, \omega \in E_i(\bar{\omega})\right\}.$$

The Lemma 3.1, stated in the previous section, allows us to ensure that the minimum is attained at every $\omega \in E_i(\bar{\omega})$, that is

$$\frac{\theta^{\ell}_{M,i}(\omega)}{p^{\ell}_{M}(\omega)} = \frac{\theta^{\ell}_{M,i}(\bar{\omega})}{p^{\ell}_{M}(\bar{\omega})},\ \text{for every } \omega \in E_i(\bar{\omega}),\ i = 1, \dots n.$$

Then, we conclude that actually $x^{\ell}_{M,i}(\omega) = \dfrac{\theta^{\ell}_{M,i}(\omega)}{p^{\ell}_{M}(\omega)}$, that is, $\theta^{\ell}_{M,i}(\omega) = p^{\ell}_{M}(\omega)x^{\ell}_{M,i}(\omega)$.

Now, we can write

$$\begin{aligned}
\frac{d_i(M)}{\|p(M)\|} &= \frac{\displaystyle\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}\theta^{\ell}_{M,i}(\omega) - \sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^{\ell}_{M}(\omega)e^{\ell}_{i}(\omega)}{\|p(M)\|} \\
&= \frac{\displaystyle\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^{\ell}_{M}(\omega)x^{\ell}_{M,i}(\omega) - \sum_{\omega\in\Omega}\sum_{\ell=1}^{L}p^{\ell}_{M}(\omega)e^{\ell}_{i}(\omega)}{\|p(M)\|} \\
&= \frac{p(M)}{\|p(M)\|}(x_i(M) - e_i).
\end{aligned}$$

Since $\|p(M)\| \to \infty$ and $d_i(M)$ is bounded for every type i, it follows that $\dfrac{p(M)}{\|p(M)\|}(x_i(M) - e_i) \to 0$, that is, $p(x_i - e_i) = 0$ for all $i \in \{1, \dots, n\}$.

Note that $px_i = pe_i > 0$, provided that $e_i \gg 0$. To finish the proof, let us show that $U_i(y) \leq U_i(x_i)$ for any bundle $y \in B_i(p)$ for every i. For it, let us take any real number $\lambda \in (0, 1)$ and a bundle $y \in B_i(p)$. Then, $\lambda py \leq \lambda pe_i = \lambda px_i < px_i$. This implies that, for all M large enough, $\dfrac{p_M}{\|p(M)\|}\lambda y < \dfrac{p_M}{\|p(M)\|}x_i(M)$ and thus $p(M)\lambda y < p(M)x_i(M) \leq M$. Let us consider the strategy given by $\alpha^{\ell}_{M}(\omega) = p^{\ell}_{M}\lambda y^{\ell}(\omega)$. Note that $\displaystyle\sum_{\omega\in\Omega}\sum_{\ell=1}^{L}\alpha^{\ell}_{M}(\omega) = p(M)\lambda y$ and then $\alpha_M \in \xi(M)$. Note that the net deficit that agent t obtains by deviating and selecting $\alpha_t = \alpha_M$ is

$$d_t(\Theta_M \backslash \alpha_t) = \left[\sum_{\omega \in \Omega} \sum_{\ell=1}^{L} \alpha_M^\ell(\omega) - p(M)e_i \right]_+ \leq [p(M)x_i(M) - p(M)e_i]_+ = d_t(\Theta_M),$$

for any $t \in I_i$. Therefore, since Θ_M is a Nash equilibrium, $U_i(x_i(M)) \geq U_i(\lambda y)$. Finally, passing to the limit and observing that $\lambda < 1$ was arbitrary, we conclude that $U_i(x_i) \geq U_i(y)$.

Q.E.D.

5 A new game: prices and common information

Consider the differential information economy $\mathcal{E}$ defined in Section 2. Let $\mathcal{P}_\mathcal{C}$ denote the common information defined as the meet of the partitions $\mathcal{P}_i, i = 1, \ldots, n$), $\mathcal{P}_\mathcal{C} = \bigwedge_{i=1}^{n} \mathcal{P}_i$[6]. If consumers could refine their information through prices, the only price systems which do not transmit any additional information to any consumer are those which are compatible with the common information structure. Indeed, by restricting prices to those which are $\mathcal{P}_C$-measurable we obtain a refinement of the Radner equilibrium solution.

In order to provide a game theoretical approach to this refinement we need to consider different games with a new price formation rule which depends crucially on the common information structure $\mathcal{P}_\mathcal{C}$. For it, we use the same notation stated in Sections 3 and 4. As in the previous game $\mathcal{G}(M)$, let us state an upper bound M on borrowing fiat money. The new game $\mathcal{G}_\mathcal{C}(M)$ is defined by the same strategy sets as $\mathcal{G}(M)$ but differs from it basically in the mechanism defining prices that is modified as follows:

Given a strategy profile $\Theta = (\theta_t, t \in I)$ and a state ω the price for the commodity ℓ in this state is given by

$$p^\ell(\omega) = \max \left\{ \frac{\theta^\ell(\omega') + 1}{e^\ell(\omega')}, \quad \omega' \in E(\omega) \right\}^7.$$

[6]The meet is the largest σ-algebra which is contained in each σ-algebra generated by $\mathcal{P}_i$, for every i. That is, $\mathcal{P}_\mathcal{C}$ is the finest partition of the set of states that is coarser than each $\mathcal{P}_i$. Note that $\{\omega\} \in \mathcal{P}_\mathcal{C}$ if and only if $E_i(\omega) = \{\omega\}$ for every i or, equivalently, information does not lead directly to any consumption restriction at the state ω for any agent.

[7]We remark that if the initial allocation e is $\mathcal{P}_\mathcal{C}$-measurable, then the price rule can be recasted as $p^\ell(\omega) = \frac{1}{e^\ell(\omega)} \left(1 + \max \left\{\theta^\ell(\omega'),\ \omega' \in E(\omega)\right\}\right)$

In this case, the amount of commodity ℓ assigned to an individual $t \in I_i$ in the state ω is given by:

$$x_t^\ell(\omega) = \min \left\{ \frac{\theta_t^\ell(\omega')}{p^\ell(\omega')},\ \omega' \in E_i(\omega) \right\} = \frac{1}{p^\ell(\omega)} \min \left\{ \theta_t^\ell(\omega'),\ \omega' \in E_i(\omega) \right\},$$

where the last equality is due to the fact that p is $\mathcal{P}_\mathcal{C}$-measurable, which implies that p is $\mathcal{P}_i$-measurable for every $i = 1, \ldots, n$.

As in $\mathcal{G}(M)$ agents do not ascribe utility to fiat money, but are penalized in the case of default. Then the payoff of each agent $t \in I_i$ for each strategy profile Θ is $\Pi_t(\Theta) = U_i(x_t(\Theta)) - d_{t+}(\Theta)$, where $x_t(\Theta)$ is the bundle previously defined and the penalizations are defined as in the previous game $\mathcal{G}(M)$.

We can easily adapt the arguments in Lemma 3.1 and Theorem 3.1, respectively, to our new game and show that the following statements hold:

> If the profile $\Theta = (\theta_t, t \in I)$ is a Nash equilibrium for the game $\mathcal{G}_\mathcal{C}(M)$, then θ_t is $\mathcal{P}_i$-measurable for almost all agent $t \in I_i$ and every type $i = 1, \ldots, n$.
>
> For every $M \in \mathbb{R}_+$ the set of symmetric Nash equilibria for the game $\mathcal{G}_\mathcal{C}(M)$ is non-empty.

In this way, for each M we obtain existence of Nash equilibrium for the game $\mathcal{G}_\mathcal{C}(M)$ in which the price formation rule leads to non-disclosure prices. In spite of this, as the next standard example shows, in general, equilibrium prices which are compatible with the common information structure does not exist.

Example 1. Let us consider an economy with three states of nature (a, b and c), one commodity in each state and three types of agents. Let x, y, z denote the consumptions in a,b and c, respectively. Agents of type 1 belong to the interval $[0, 1/3)$ and are characterized by the partition $\{\{a\}, \{b, c\}\}$, the utility function $U_1(x, y, z) = x + 2y + 2z$ and the initial endowments $e_1 = (1, 1/2, 1/2)$. Agents of type 2 belong to the interval $[1/3, 2/3)$ and are characterized by the partition $\{\{a, b\}, \{c\}\}$, the utility function $U_2(x, y, z) = 2x + 2y + z$ and the initial endowments $e_2 = (1/2, 1/2, 1)$. Agents of type 3 belong to the interval $[2/3, 1]$ and are characterized by the partition $\{\{a\}, \{b\}, \{c\}\}$, the utility function $U_3(x, y, z) = x + 2y + z$ and the initial endowments $e_3 = (1/2, 1, 1/2)$. The unique non-disclosure price system satisfies $p(a) = p(b) = p(c)$ and leads to the demanded bundles $(0, 1, 1)$, $(1, 1, 0)$ and $(0, 2, 0)$, for each type 1, 2 and 3, respectively. These bundles results in a non-feasible allocation. Therefore, in this economy there is no equilibrium price that does not transmit information to consumers.

Therefore, we need additional assumptions in order to obtain equilibrium with non-disclosure prices as a limit of a sequence of prices and allocations which results from a sequence of Nash equilibria. Actually, if the total endowment e is $\mathcal{P}_{\mathcal{C}}$-measurable and preferences verify a risk aversion property[8], then we can show existence of this refinement of equilibrium. The required assumption on preferences means that a uniform consumption across the states that are not commonly distinguished is preferred to a more diversified bundle. We remark that this condition may be verified not only by different expected utility functions but also for other types of preferences[9].

The previous assumptions on endowments and preferences allow us to adapt the proof of Theorem 4.1 and obtain the existence of non-disclosure equilibrium prices as follows:

> For each integer M, we can take $\Theta_M = (\theta_{M,t},\ t \in I)$ which is a symmetric Nash equilibrium for the game $\mathcal{G}_{\mathcal{C}}(M)$, such that $\theta_{M,t}$ is $\mathcal{P}_C$-measurable for almost all $t \in I$. Let $(p(M), x(M))$ be the corresponding sequence of prices and allocations which is defined by this sequence of Nash equilibria. Then, there exists a subsequence of $(p(M)/\|p(M)\|, x(M))$ which converges to a price system p which is $\mathcal{P}_{\mathcal{C}}$-measurable and an allocation x, such that (p, x) is an equilibrium for the economy $\mathcal{E}$.

We conclude this Section presenting an example which shows not only that the required assumption on preferences is not a necessary condition for the above existence result but also that the corresponding equilibrium bundles need not to be measurable with respect the common information structure.

[8]For every consumer $t \in I,\ U_t(\mathcal{A}(x)) \geq U_t(x)$, where $\mathcal{A}(x)(\omega) = \dfrac{\sum_{\omega \in E(\bar{\omega})} x(\omega)}{Card(E(\bar{\omega}))}$, for each $\omega \in E(\bar{\omega})$, being $E(\bar{\omega})$ the common information event which contains $\bar{\omega}$.

[9]Precisely, the assumption holds for expected utility functions defined by a state dependent utility which is concave and constant across common information events, whenever priors are compatible with the common information structure (for instance, every state is equally probable). Moreover, our requirement still holds for other types of preference relations.

Let $U(x) = \sum_{E \in \mathcal{P}_C} \min\{u(x(w)), w \in E\}$, where u is a concave function. U satisfies the requirement on preferences and is related with a conservative behavior.

Example 2. Consider an economy with three consumers, three states of nature a, b and c and one commodity in each state. Let x, y, z denote the consumptions in a,b and c, respectively. Consumer 1 is characterized by the information $\{\{a\}, \{b, c\}\}$, utility function $U_1(x, y, z) = x^2yz$ and endowments $\omega_1 = (1, 0, 0)$ Consumer 2 is characterized by the information $\{\{b\}, \{a, c\}\}$, utility function $U_2(x, y, z) = xy^2z$ and resources $\omega_1 = (0, 1, 0)$. Consumer 3 is characterized by the informaation $\{\{c\}, \{a, b\}\}$, utility function $U_3(x, y, z) = xyz^2$ and endowments $\omega_1 = (0, 0, 1)$. This economy has a non-revealing equilibrium given by prices $p_a = p_b = p_c = 1$ and the bundles $(1/2, 1/4, 1/4)$ for the consumer 1, $(1/4, 1/2, 1/4)$ for the consumer 2 and $(1/4, 1/4, 1/2)$ for the consumer 3.

6 Concluding Remarks

We have followed a market game approach à la Shapley-Shubik in order to analyze the equilibrium price formation in our differentiated information market. Agents deliver to each trading-post their endowments for sale and the game starts. The strategic variable of each player is the amount of fiat money that she wants to spend for each commodity in each state. The price arises in each post according to the rule of supply and demand. Once the price system is formed, the ratio strategy-price could be different in states that an agent does not distinguish. However, the game allocates the minimum and therefore agents are never deceived. The rules of the game underlie a conservative or prudent behavior of the consumers. Then, this market game leads to a Radner equilibrium without the explicit assumption that agents choose constant consumption in states which they do not distinguish. Moreover, our result can be used in order to provide a strategic market game support to the prudent behavior of the agents considered in Correia da Silva and Hervés-Beloso (2009) and De Castro and Yannelis (2010).

On the other hand, we have provided a new existence result of equilibrium with non-disclosure prices. The essence of this refinement is that the underlying equilibrium prices transmit no additional information on the states of nature to agents and therefore there is no room to learn from prices.

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